Emergence of binary tree networks and their inevitable self-similar topology

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We show that in open dissipative systems evolutionary dynamics driven by a flow gradient and subject to proximity constraint, that is, the matter and energy can traverse only through a continuum, in the presence of inherent randomness, give rise to a tree topological organization. Moreover, we argue that the observed optimality in energy expenditure is merely a consequent signature of the formation of tree networks instead of the cause. Self-similar topology, which can be characterized as power law size distribution, has been found in diverse tree networks ranging from river networks to taxonomic trees. We find that the statistical self-similar topology is an inevitable consequence of any full binary tree organization. We show this by coding a binary tree as a unique bifurcation string. This coding scheme allows us to investigate trees over the realm from deterministic to entirely random trees. To obtain partial random trees, partial random perturbation is added to the deterministic trees by an operator similar to that used in genetic algorithms. Our analysis shows that the hierarchical density of binary trees is more diverse than has been described in earlier studies. We find that the connectivity structure of river networks is far from strict self-similar trees.